$$
\begin{aligned}
& \text { HWO } \\
& \text { SETS }
\end{aligned}
$$

Topic 0 - Sets
Def: A set is a collection of objects. The objects in the set are called the elements of the set.
If $S$ is a set and $x$ is an element of the set $S$ then we write $x \in S$.

If $x$ is not an element of $S$ then we write $x \notin S$.

Ex:

$$
\begin{aligned}
& S=\left\{10, \frac{1}{2}, e, 4,-\sqrt{2}\right\} \\
& e \in S \\
& \frac{1}{3} \notin S
\end{aligned}
$$

Note: In a set, order doesn't matter. So, for example

$$
\begin{aligned}
& S=\left\{10, \frac{1}{2}, e, 4,-\sqrt{2}\right\} \\
& \\
& =\left\{\frac{1}{2},-\sqrt{2}, 4, e, 10\right\}
\end{aligned}
$$

Note: In a set you can't have duplicates. So, $\{1,1,2\}$ isn't a set.
If someone wrote $\{1,1,2\}$ then they really mean $\{1,2\}$

Ex: $\mathbb{R}$ denotes the set of real numbers. The real numbers are the numbers on the number line with decimal expansions.


$$
\begin{aligned}
& \mathbb{R}=\left\{0,1,-3, \frac{1}{2}, 0.673, \pi,\right. \\
& e, \sqrt{2}, \ldots .0\} \\
& \frac{\pi}{10} \approx 0.314159 \ldots \in \mathbb{R} \\
& -10,000 \in \mathbb{R} \quad \begin{array}{c}
\text { infinitely } \\
\text { many } \\
\text { more }
\end{array} \\
& \sqrt{-3} \notin \mathbb{R} \quad\left[\text { Here } i^{2}=-1\right]
\end{aligned}
$$

Notation:
$x, y \in S$ is shorthand for $x \in S$ and $y \in S$.
read: " $x$ is in $S$ and $y$ is in $S^{\prime \prime}$

Ex: $10, \sqrt{2} \in \mathbb{R}$
shorthand fir: $\sqrt{10} \in \mathbb{R}$ and $\sqrt{2} \in \mathbb{R}$

Ex: $0,1, \frac{1}{2},-\sqrt{2} \in \mathbb{R}$
shorthand for: $0 \in \mathbb{R}, \mid \in \mathbb{R}$,

$$
\frac{1}{2} \in \mathbb{R},-\sqrt{2} \in \mathbb{R}
$$

General way to define a set

$$
\left\{\begin{array}{l|l}
\text { description of } & \begin{array}{l}
\text { condition that } \\
\text { the elements } \\
\text { What the look } \\
\text { elements look } \\
\text { must satisfy to } \\
\text { be in the set }
\end{array}
\end{array}\right\}
$$

some people use - instead of 1
read I as "such that" or "where"

Ex:

$$
S=\{(x, y) \mid x \in \mathbb{R} \text { and } y \in \mathbb{R}\}
$$

could also write as " $x, y \in \mathbb{R}^{\prime \prime}$
read: $S$ consist, of all $(x, y)$ where $x$ and $y$ we real numbers

$S$ is the $x y$-plane

Def: The empty set, denoted by $\phi$ or $\}$, is defined to be the set with no elements.

Def: Let $A$ and $B$ be sets. We say that $A$ is a subset of $B$ if every element of $A$ is also an element of $B$.


If $A$ is a subset of $B$ then we write $A \subseteq B$
$E x:$

$$
\begin{aligned}
& A=\{6,4,789\} \\
& B=\left\{3, \frac{1}{2}, 6, \pi, 789,4\right\}
\end{aligned}
$$

Then $A \subseteq B$


